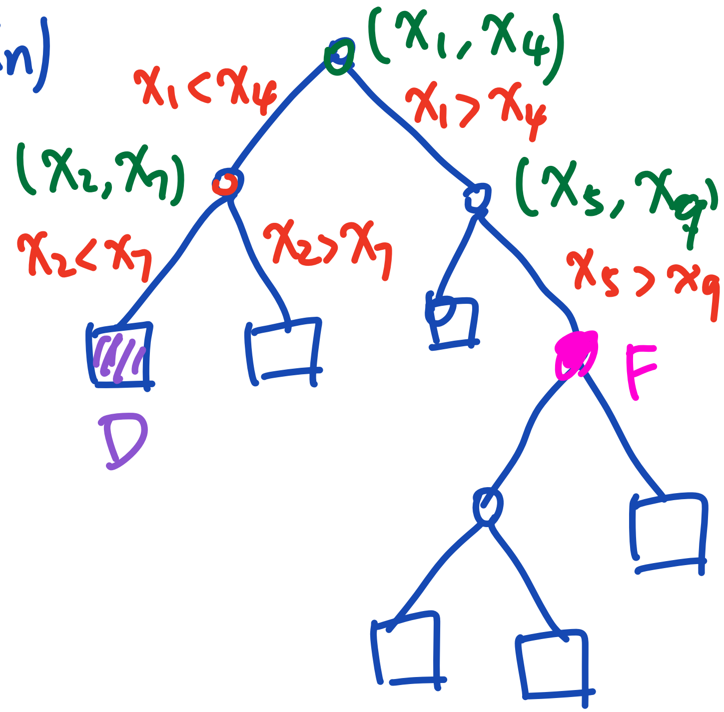
$(x_1, x_2, \cdots, x_n)$



$D = \{ \vec{x} \in \mathbb{R}^n : x_1 < x_4$ 且 $x_2 < x_7 \}$

$F = \{ \vec{x} \in \mathbb{R}^n : x_1 > x_4$ 且 $x_5 > x_9 \}$

输入：$\vec{x} = (x_1, x_2, \cdots, x_n) \in \boxed{\mathbb{R}^n}$

内部结点： (x_i, x_j) 比较

边.

结点：$\mathbb{R}^n$ 子集

$D_1, D_2, \cdots, D_k$ 叶结点

$D_i \cap D_j = \phi$

$\bigcup_{i=1}^{k} D_i = \mathbb{R}^n$

$\boxed{A_1, A_2, \ldots, A_\ell}$

$k \geq \ell$.

<u>Search</u>.

Input: $\vec{x} = (x_1, x_2, \cdots, x_n)$ $y \in \mathbb{R}$.

Output: $i \in \underline{[n]}$, s.t. $x_i = y$

叶结点个数 $\geq n$ $\Omega(\log n)$

Sorting:

Input: $\vec{x} = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$

Output: sorted list

Consider $\forall \sigma \in S_n$ $\qquad |S_n| = n!$

$A_\sigma = \{\vec{x} \in \mathbb{R}^n : x_{\sigma(1)} < x_{\sigma(2)} < \cdots < x_{\sigma(n)}\}$

Claim. $\forall \sigma, \tau \in S_n$, $\sigma \neq \tau$, 则 A_σ, A_τ 进入不同叶结点.

$\Rightarrow$ 叶结点个数 $\geq |S_n| = n!$

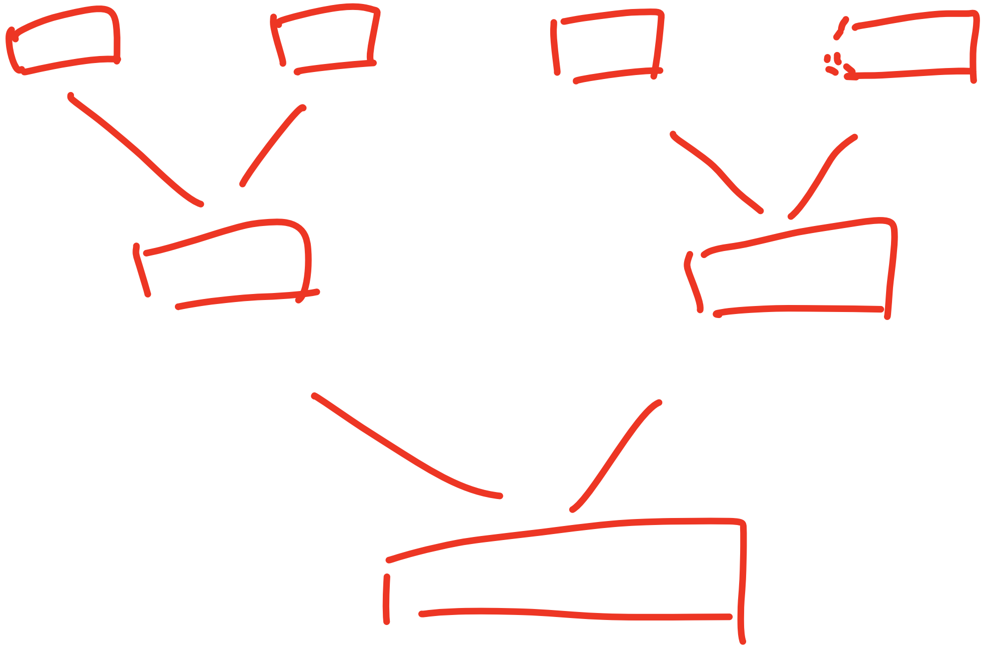
Sorting.

Input: $\vec{x} = (x_1, \cdots, x_n) \in \mathbb{R}^n$

Output: $(\underline{i_1}, \underline{i_2}, \cdots, i_n)$. s.t. $x_{i_1} < x_{i_2} < \cdots < x_{i_n}$

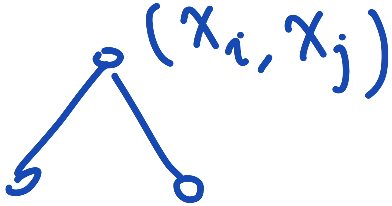
输出个数 $= n!$ 种

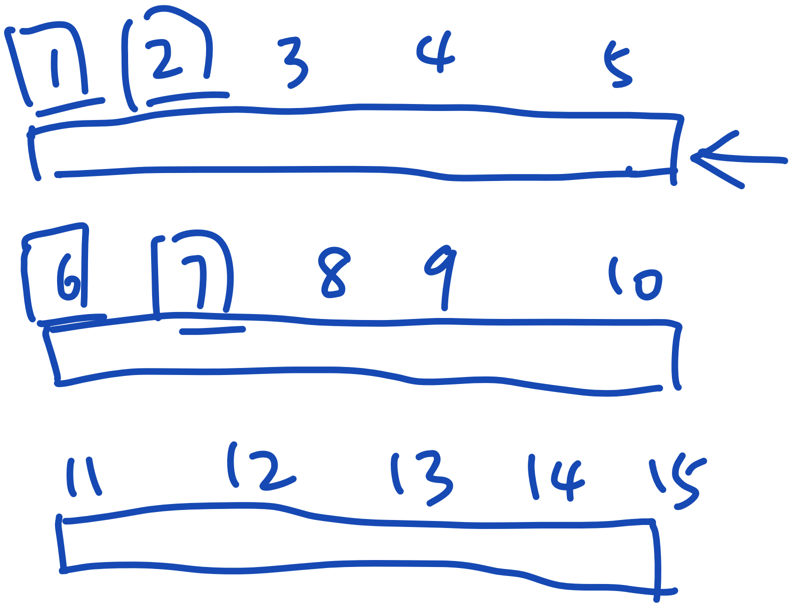
$\Rightarrow$ 叶结点 $\geq$ 输出个数 $\geq n!$



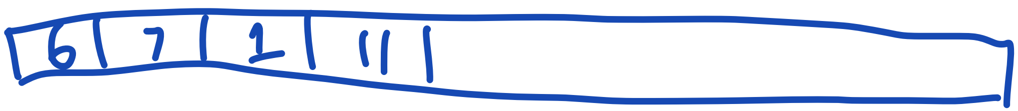
$$T(n,k) = T\left(n, \frac{k}{2}\right) + O(n)$$

$$\Rightarrow \quad T(n,k) = O(n \log k)$$





$$\binom{n}{\frac{n}{k}, \frac{n}{k}, \dots, \frac{n}{k}} = \frac{n!}{\left(\left(\frac{n}{k}\right)!\right)^k}$$



$$\log(\quad) = \Omega(n \log k)$$

$\Theta(n\log n)$ $\begin{cases} O(n\log n) \\ \Omega(n\log n) \end{cases}$

决策树

$\log(n!) = \Theta(n\log n)$

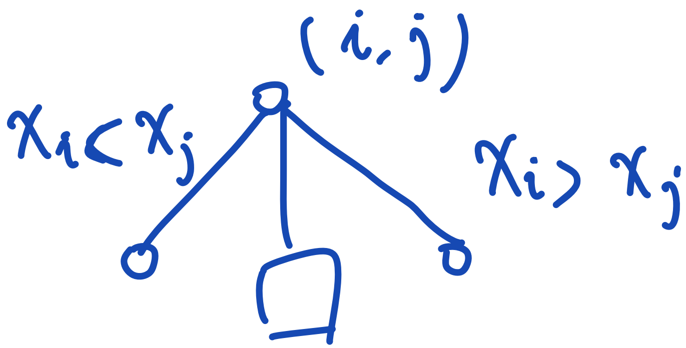
World 1 $\{=, \neq\}$

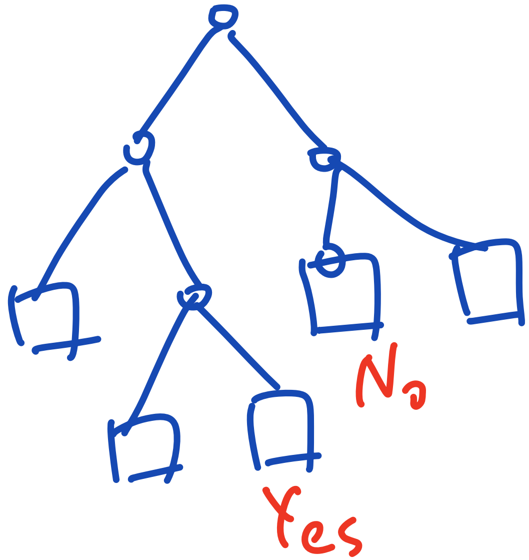
上界: $O(n^2)$

下界: $\cancel{\Omega(n\log n)}$ $\Omega(n^2)$

World 2. $\{=, <, >\}$

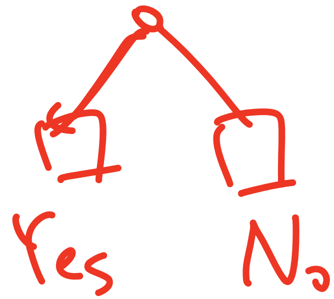
上界: $O(n\log n)$





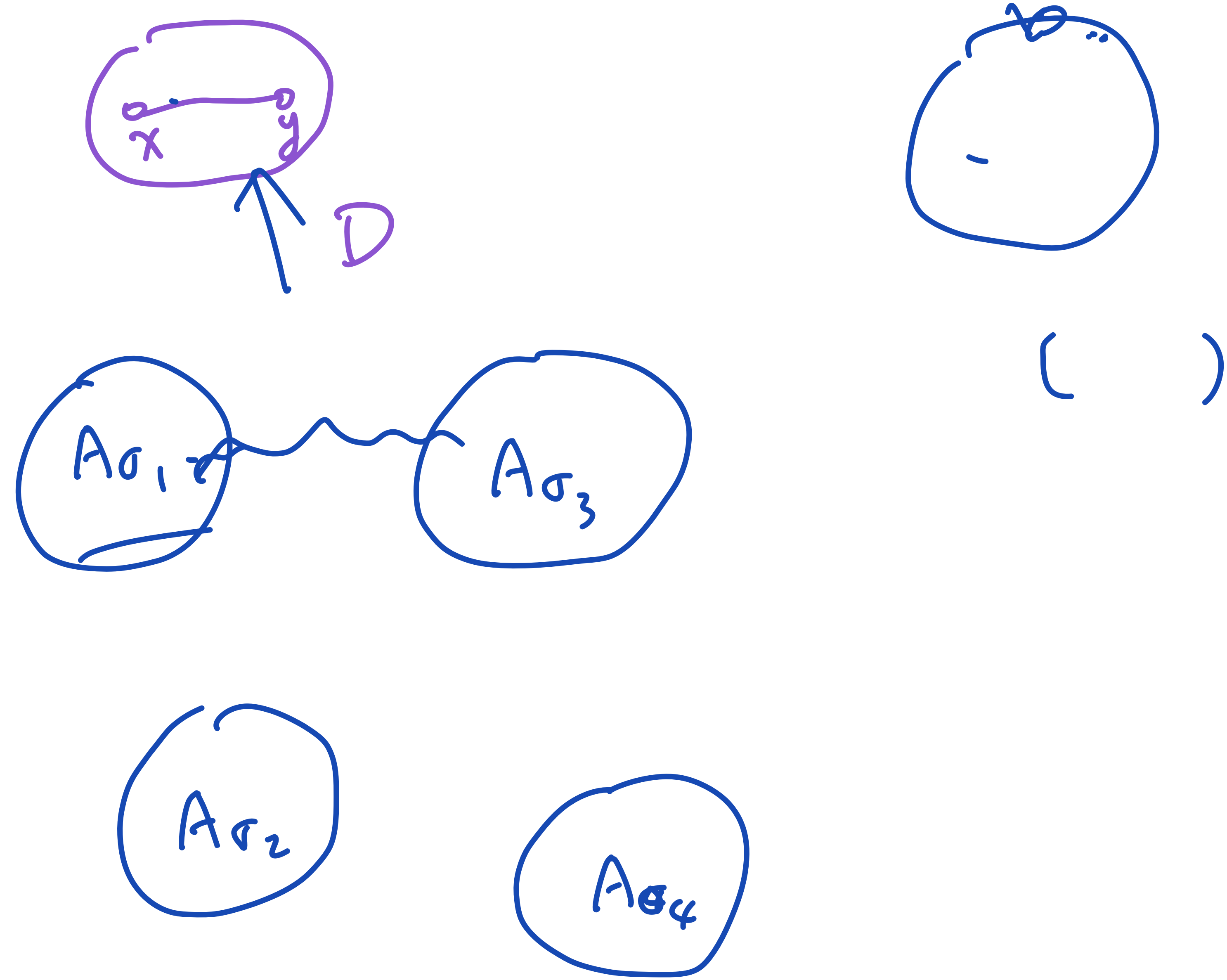
$(1, 2, 3, 4, 5)$

$\underbrace{(5, 3, 1, 4, 2)}$



$(x_1, x_2, \cdots, x_n)$ $\qquad$ $x_i \longleftrightarrow x_j$

$$\begin{cases} (x_1, x_2, \cdots, x_i, \cdots, x_j, \cdots, x_n) \longrightarrow \text{Yes} \\ (x_1, x_2, \cdots, x_i, \cdots, x_i, \cdots x_n) \longrightarrow \text{No} \end{cases}$$



$$A_\sigma = \{\vec{x} \in \mathbb{R}^n : x_{\sigma(1)} < x_{\sigma(2)} < \cdots < x_{\sigma(n)}\}$$